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MAGNETOCONDUCTANCE OF AN n-TYPE Si INVERSION LAYER IN THE TRANSITION REGION OF WEAK TO STRONG LOCALIZATION

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The anomalous transverse magnetoconductance of an n-type (100)Si inversion layer has been investigated at temperatures between 1.4 and 4.2 K for electron densities below $9 \times 10^{15}/m^2$. A general dependence of $\Delta\sigma/\sigma(B=0)$ on the ratio of the thermal length $L_T = (D\hbar/kT)^{1/2}$, D being the diffusion coefficient, and the magnetic length $L_B = (\hbar/eB)^{1/2}$ is found to hold up to at least 4.5 tesla if $k_F l \gtrsim 2$. Replacing L_B by a magnetic diffusion length $L_c = (D/\omega_c)^{1/2}$, ω_c being the cyclotron frequency, the same dependence is observed to hold for $k_F l < 2$, but is restricted to weak fields due to the onset of a negative contribution to $\Delta\sigma/\sigma(B=0)$ which is closely related to the transition to strong localization.

1. Introduction

Ever since the observation of a transition of thermally activated to quasi-metallic conductance in the two-dimensional system of electrons confined to a MOS-inversion layer, unceasing efforts have been made to describe electronic transport in this region [1]. According to the scaling theory of Abrahams et al. [2] and Anderson et al. [3], the conductance should gradually change from an exponential to a logarithmic dependence on length scale, when the wavelength $1/k_F$ becomes of the order of the electronic mean free path l. Thus for a (100)Si inversion layer with two-fold valley degeneracy, the conductivity σ at the transition is expected to be about $e^2/\pi\hbar (\simeq 7.7 \times 10^{-5}$ mho). The subsequent microscopic treatment of transport properties in the regime of weak localization ($k_F l \gg 1$) by Hikami et al. [4], along with the investigation of effects contributed by mutual electron interactions by Altshuler et al. [5] and Fukuyama [6], enabled an accurate quantitative comparison to be made with both the observed logarithmic temperature corrections to the conductivity and the associated anomalous magnetoconductance. These aspects of two-dimensional electron transport have now, at least qualitatively, been verified in numerous experiments in the case of strictly weak localization [7], although much of the experimental evidence can alternatively be understood in terms of power-law localization [8].

Magnetotransport has hardly been investigated in the transition region to strong localization [9], probably due to the lack of adequate knowledge of how localization would work out in this intermediate region. Applying the weak localization expression for the transverse magnetoconductance in the quasi-metallic limit, which was first derived by Hikami et al. [4], to experimental results even below $k_F l = 1$, Kawaguchi and Kawaji observed a sharp transition in electronic transport for $k_F l$ just above 1. These authors assumed there is a close relation to valley splitting as had been proposed by Bloss et al. [10] some time before, which is however, expected to occur at a still lower electron density, specifically below $5 \times 10^{15}/\mathrm{m}^2$.

In this study a transition is substantiated on the basis that the magnetoconductance in the region where $k_F l \approx O(1)$, is found to be distinct from the predicted weak localization behaviour when $k_F l \gg 1$. The present magnetoconductance results have been obtained from a low-mobility device of which the temperature dependence of the conductivity was previously investigated [11,12] in the same region of $k_F l$ and for comparable experimental details. The anomalous increase $\Delta\sigma$ of the conductance due to a perpendicularly applied magnetic field will be demonstrated to be proportional to the conductivity in zero field and to depend on length scales, that are governed by the magnetic field and temperature.

2. The conductivity in transverse magnetic fields

2.1. Dependence on the temperature

In order to establish how far the electron density could be lowered without the distinctive logarithmic temperature corrections for weak localization disappearing, the temperature dependance of the conductivity was re-examined between 1×10^{-4} mho and 1×10^{-5} mho. In fig. 1 it can be seen, that for the electron density as low as $6 \times 10^{15}/\mathrm{m}^2$ and the temperature between 1.3 and 4.2 K, the conductivity surprisingly exhibits an apparent logarithmic temperature dependence, which however, can no longer be considered as a small correction to the Boltzmann term. Nevertheless, the logarithmic contribution being independent of electron density and having a slope of 1.55 in units $e^2/2\pi^2\hbar$, might be regarded as a rather reliable argument to support a description in terms of both localization and interaction in the limit of $k_F l \gg 1$ [6] still being applicable, irrespective of the relative contribution provided by either of the two mechanisms.

To illustrate the effect of the temperature on the positive magneto-conductance, its relative change $\Delta\sigma/\sigma(B=0)$ is shown in fig. 2 as a function of the applied magnetic field for various temperatures, at an electron density which is representative for $\sigma(B=0)$ in the range between 2×10^{-5} and 8×10^{-5} mho.

A cursory analysis might give the impression that the curves exhibit the characteristics currently observed at higher electron densities [7], where $k_F l \gg 1$. This seems to be indicated by a parabolic onset at low strength of the field, fading into a logarithmic dependence above the inflection point of each curve. One might even be fortified in that opinion by the fact that the measured dependence of $\Delta\sigma = \sigma(B) - \sigma(B=0)$ on the magnetic field was always found to be well-fitted for weak fields by the theoretical expression of Hikami et al. [4]:

$$\Delta\sigma = \alpha g_v \frac{e^2}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2}+\frac{1}{\tau_\epsilon a}\right)+\ln(\tau_\epsilon a)\right],$$

where $a = (4/\hbar)DeB$ (D denoting the diffusion constant), τ_ϵ is an inelastic scattering time, g_v the valley degeneracy, and α an empirical prefactor depending on the type of valley scattering.

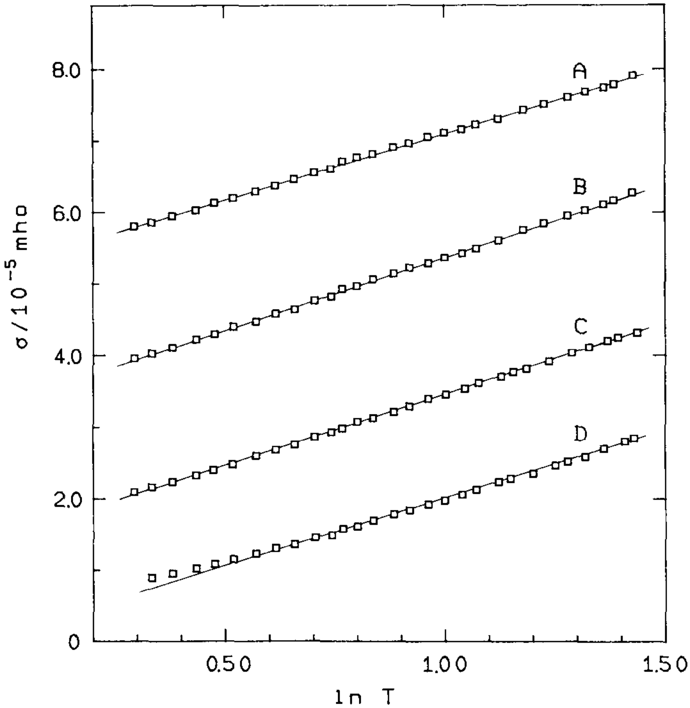

Fig. 1. Logarithmic temperature contribution to the conductivity in the region of weak localization. The values of the gate voltage and corresponding electron density are: (A) 1.48 V, $7.6\times10^{15}/m^2$; (B) 1.26 V, $7.1\times10^{15}/m^2$; (C) 1.00 V, $6.5\times10^{15}/m^2$; (D) 0.75 V, $5.9\times10^{15}/m^2$.

However, the factor α then appears to be forced to vary almost proportionally with the conductivity $\sigma(B=0)$ to account for the strong variation of the observed magnetoconductance $\Delta\sigma$ with the electron density or temperature.

This induced property of the prefactor shows that extrapolation of any theoretical weak localization result to the region where $k_F l$ becomes of the order of unity, will generally be incorrect. Although a fit by the above expression of Hikami et al. might be strongly anticipated from the logarithmic temperature dependence of the conductivity, the finding of a prefactor proportional to $\sigma(B=0)$ is so contradictory, that only the appearance of a relative change of the magnetoconductance can be retained instead of an absolute variation expressed in units of $e^2/2\pi^2\hbar$.

Examination of the slope S_T of the inflectional tangents of the curves in fig. 2 reveals that it varies inversely with the temperature, as is shown in fig. 3 by

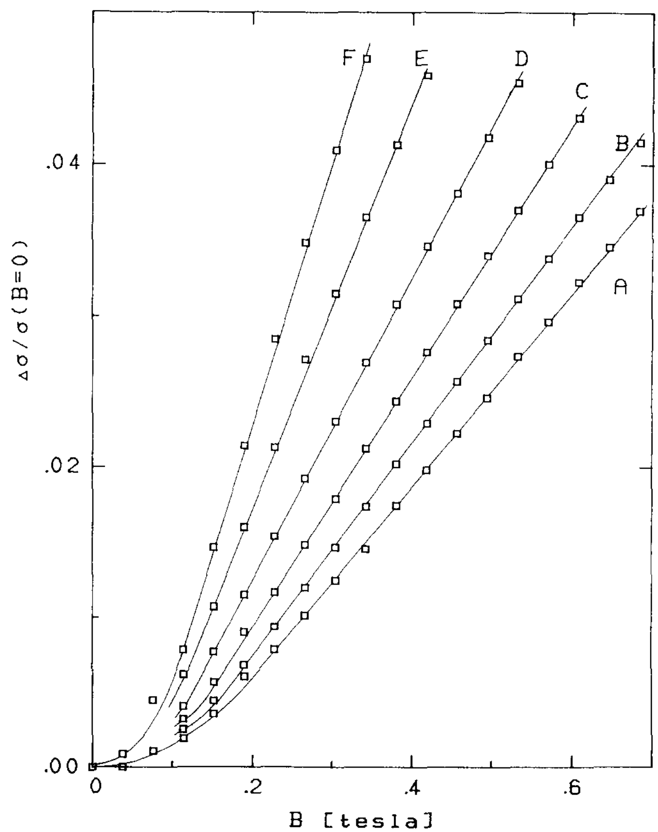

Fig. 2. Relative increase of the conductivity in a perpendicular magnetic field for $V_g = 1.25$ V at several temperatures: (A) 4.00 K; (B) 3.50 K; (C) 3.00 K; (D) 2.50 K; (E) 1.92 K; (F) 1.49 K.

the linear dependence of $1/S_T$ on the temperature extrapolating to $1/S_T = 0$ at $T = 0$. By plotting $\Delta\sigma/\sigma(B=0)$ versus B/T, it can now be easily verified that the relative magnetoconductance varies with B/T over the whole range of the magnetic field and the temperature considered so far.

2.2. Dependence on the Fermi level

Similar behaviour was observed in the curves of $\Delta\sigma/\sigma(B=0)$ versus B when the gate voltage was varied, but so as to keep the electron density below $n_s = 9 \times 10^{15}/\mathrm{m}^2$. Above this value the magnetoconductance presented a rather sharp change over to the predicted absolute type of correction, in precisely the same way as was previously observed by Kawaguchi and Kawaji [9]. The latter type will be discussed elsewhere, also taking electron–electron interaction into account. The relation that results between the applied gate voltage V_g, or equivalently the Fermi level ϵ_F, and the slope S_V of the corresponding inflectional tangent is illustrated in fig. 4 for the range of the electron density considered at $T = 4.23$ K. Here it should be noted that S_V extrapolates to zero for $V_g = -1.85 \pm 0.05$ V, where the Fermi level is just at the bottom ϵ_0 of the

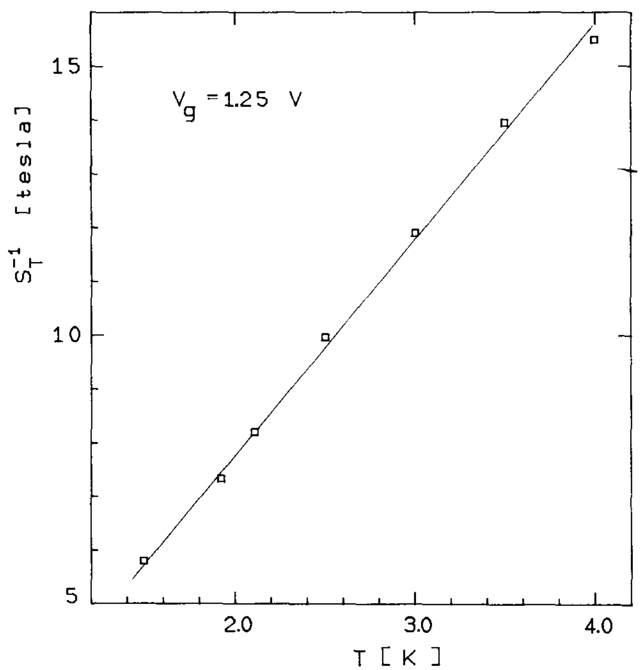

Fig. 3. The reciprocal slope of the linear part in the curves of $\Delta\sigma/\sigma(B=0)$ versus B, as shown in fig. 2, plotted as a function of the temperature for $V_g = 1.25$ V.

lowest subband, as is known from capacitance measurements as well as from the analysis of the periods of Shubnikov–De Haas oscillations. Because of the linear dependence of S_V on the gate voltage, one is now prompted to examine the variation of $\Delta\sigma/\sigma(B=0)$ as a function of $B(\epsilon_F - \epsilon_0)$, as a result of which all relative magnetoconductance curves belonging to a single temperature are found to coincide.

2.3. Length scales

By combining the above results it follows that $\Delta\sigma/\sigma(B=0)$ can be described by a general function f of an independent variable having the form $\mu B(\epsilon_F - \epsilon_0)/kT$, in which the "mobility factor" μ has been introduced as a constant by dimensional argument.

This dimensionless variable can be regarded as the ratio of two fundamental length scales L_T and L_B which are defined by the diffusion length

$$L_T = (D\hbar/kT)^{1/2},$$

D being the diffusion constant $\mu(\epsilon_F - \epsilon_0)/e$, and the radius of the ground

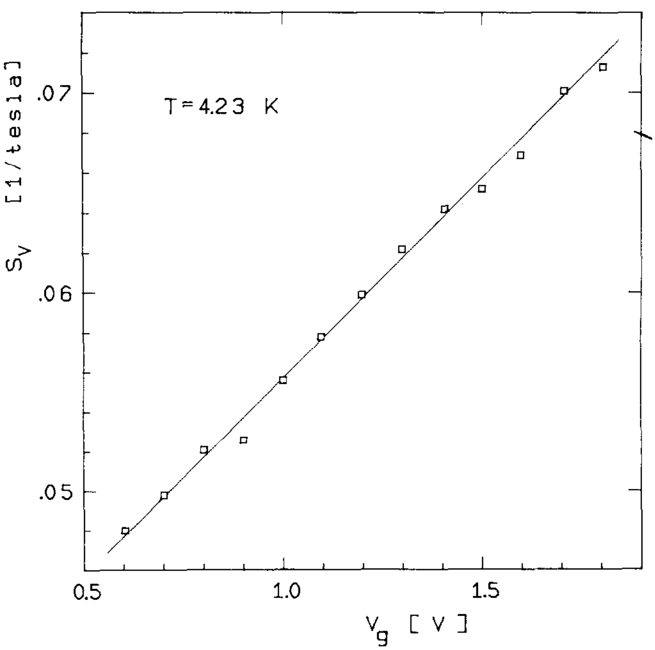

Fig. 4. The slope of the linear part in the curves of $\Delta\sigma/\sigma(B=0)$ versus B, as a function of the gate voltage for $T = 4.23$ K.

Landau orbit

$$L_B = (\hbar/eB)^{1/2}.$$

The length L_T is also thought to be the effective scale in the argument of the observed logarithmic temperature correction shown in fig. 1.

As was argued by Kaveh et al. [13], further lowering of the Fermi level, which in the present study of low-mobility devices also involves the transition

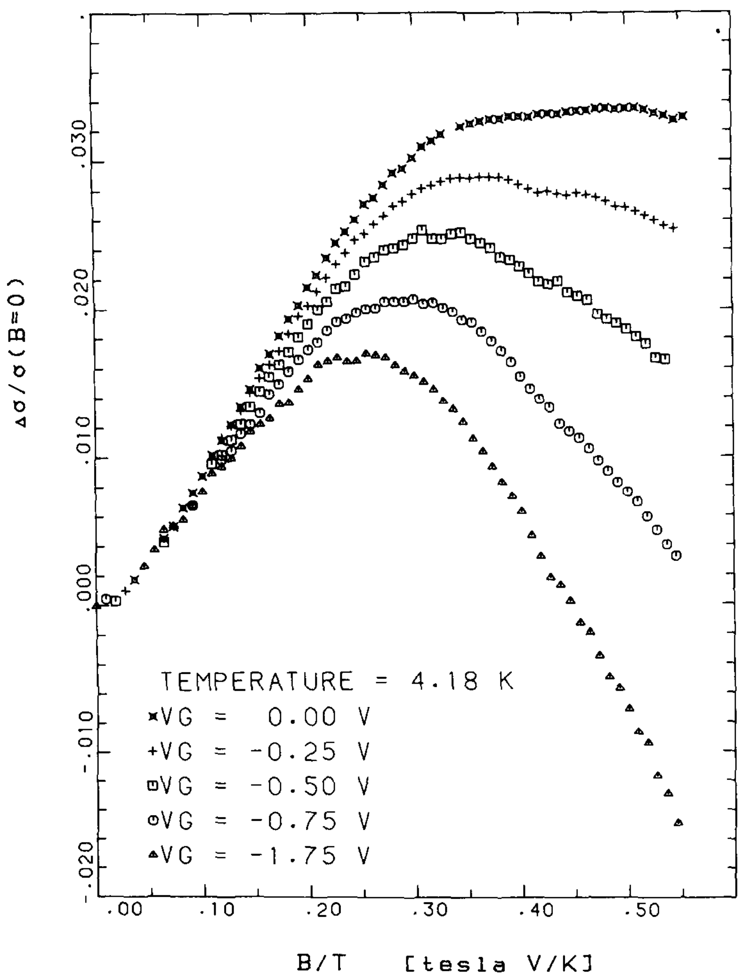

Fig. 5. The relative magnetoconductance as a function of B/T, showing the absence of a dependence on the Fermi energy in low fields for $k_F l < 2$.

to thermally activated conductance [12], will make the magnetic length scale L_c, defined in terms of the cyclotron frequency ω_c by

$$L_c = (D/\omega_c)^{1/2},$$

smaller than L_B.

This transition from L_B to L_c, occurring at $\lambda = (\epsilon_F - \epsilon_0)\tau/\hbar = 1$, can be easily identified, as $\Delta\sigma/\sigma(B=0)$ will then become a function of $\hbar\omega_c/kT$ and will therefore no longer depend on the Fermi level. The expected absence of any effect of ϵ_F (or V_g) on $\Delta\sigma/\sigma(B=0)$ is confirmed in the region of thermally activated conductivity, as is shown in fig. 5 for several gate voltages between 0 and -1.75 V at $T = 4.18$ K, but appears to be restricted to ever lower fields when the strength of localization increases. A competing negative contribution to the magnetoconductance typically shows up in passing below $\lambda = 1$. This indirect reference to a possibly sharp transition from weak to strong localization will be the subject of further investigation and will not be discussed here.

2.4. Discussion

Even though a theoretical treatment is at present unfortunately not available, it seems nevertheless reasonable to expect that the function $f(L_T^2/L_B^2)$ will describe the observed relative magnetoconductance equally well below $\lambda = 1$, if L_B is replaced by L_c, regardless of the fact that the explicit form of the function itself is still unknown. Due to this change of the magnetic length from L_B to L_c the mobility can be determined simply by scaling both experimental curves of $\Delta\sigma/\sigma(B=0)$ versus $B(\epsilon_F - \epsilon_0)/T$ respectively B/T at sufficiently low fields so as to make them fall together in a single curve. In this way, the mobility was found to be 0.20 ± 0.01 m^2/V · s, yielding $\lambda = 1.0 \pm 0.1$ at the estimated "transition voltage" of 0.25 V.

In this context it may be worthwhile dwelling on the fact that almost the same value of the mobility was previously obtained from an analysis of the thermally activated conductivity in terms of a mobility edge model [12]. The absolute temperature corrections when $\lambda > 1$ were ascribed, at the time, to a temperature dependence of the Fermi level [11], but are at present rather related to weak localization and mutual electron interactions. These logarithmic corrections as shown in fig. 1 comprise, according to the present point of view, a localization contribution and an interaction term, both having the form $\ln(\hbar/\tau kT)$, and are thus consistent with a mobility independent of the gate voltage and the temperature, as the increase of the conductivity with the gate voltage then yields $\mu = 0.20 \pm 0.01$ m^2/V · s, when taking 1.58×10^{15}/m^2 · meV for the density of states. This value is in agreement with the previous result.

The question that now arises is why the apparent metallic diffusion is not affected, even if the experimental conditions approach strong localization. In an attempt to find any delocalization effect on the mobility in the region where $\lambda > 1$, the magnetoconductance was measured up to fields as large as 4.5 tesla, but nevertheless found too weak for saturation of $\Delta\sigma/\sigma(B=0)$ to be attained. Some of the high-field data are represented in fig. 6, where $\Delta\sigma/\sigma(B=0)$ has been plotted for several gate voltages and temperatures as a function of $B(V_g - V_{th})/T$ on a logarithmic scale. The above description of the relative magnetoconductance now appears to be valid even in strong fields, without the diffusion constant being noticeably distorted.

It also appears that $\Delta\sigma/\sigma(B=0)$ closely approaches a logarithmic dependence on L_T^2/L_B^2, which on extrapolation and adopting $\mu = 0.20\ m^2/V\cdot s$, is found to intersect the horizontal logarithmic axis for $\mu B(\epsilon_F - \epsilon_0)/kT$ close to 1. Thus, this logarithmic asymptote to the function $f(L_T^2/L_B^2)$ in connection with the above logarithmic temperature correction of the conductivity might bring to mind the power-law localization proposed by Kaveh [8]. However, the description of the magnetoconductance according to this theory does not fit the observed quadratic dependence of $\Delta\sigma/\sigma(B=0)$ on the magnetic field very

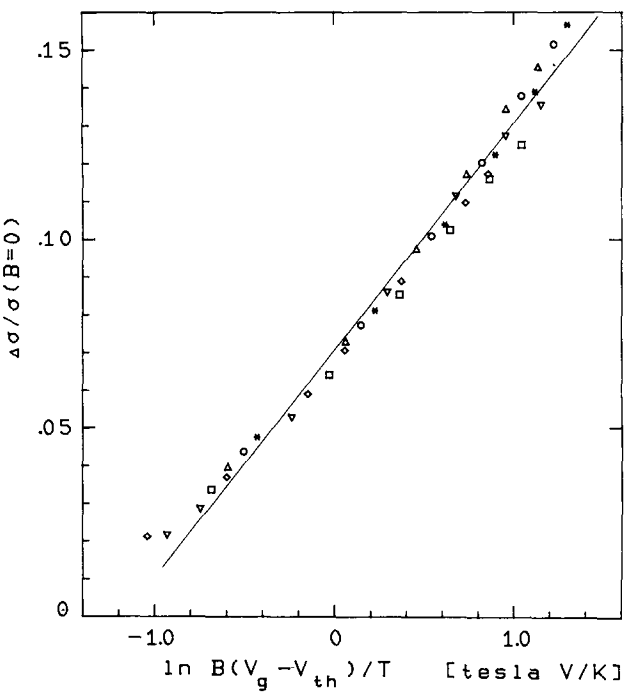

Fig. 6. The logarithmic behaviour of $\Delta\sigma/\sigma(B=0)$ in strong magnetic fields, showing the general dependence on L_T^2/L_B^2 for $k_F l > 2$: (∗) 1.50 V, 4.23 K; (○) 1.25 V, 4.23 K; (△) 1.00 V, 4.23 K; (□) 0.75 V, 4.23 K; (▽) 1.25 V, 1.49 K; (◇) 1.25 V, 3.00 K.

well when $L_T \lesssim L_B$ and, contrary to present observations, involves a logarithmic factor $\hbar/(\epsilon_F - \epsilon_0)\tau$, when expanded into a series of logarithmic corrections.

An alternative interpretation spanning the whole region of quasi-metallic and thermally activated conductivity was given by Gold [14] on the basis of a self-consistent current relaxation theory. A definite proof of the merits of this approach must unfortunately await the accompanying anomalous magnetoconductance effects being dealt with as well.

3. Conclusions

It has been demonstrated that the applicability of electronic transport properties of weakly localized electrons cannot be extrapolated down to $k_F l \approx 0(1)$. Experimental evidence has been adduced in support of a novel form of anomalous transverse magnetoconductance with $\Delta\sigma = \sigma(B) - \sigma(B=0)$ proportional to $\sigma(B=0)$. The characteristic magnetic length scale $(\hbar/eB)^{1/2}$ or $(D/\omega_c)^{1/2}$, whichever is smallest depending on the Fermi level, and the thermal length $(D\hbar/kT)^{1/2}$ have been shown by phenomenological reasoning to govern the relative magnetoconductance $\Delta\sigma/\sigma(B=0)$ in the transition region from weak to strong localization.

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